

EXPERIMENTAL AND THEORETICAL RESULTS FOR MASS TRANSFER TO A ROTATING CONE IN A NON-NEWTONIAN SALINE SOLUTION

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Abstract—A theoretical expression has been obtained for the heat or mass transport to a rotating cone in a non-Newtonian fluid. The fluid used was an aqueous sodium chloride solution which was rendered non-Newtonian by adding Polyox WSR 301. Experimentally measured mass-transfer rates for a 60° cone have been compared to the theoretical prediction and good agreement has been obtained for four polymer solutions. For the highly viscous solution a secondary flow has been observed at moderate rotational speeds.

NOMENCLATURE

a ,	constant;	Re_L ,	Reynolds number defined in equation (15);
c ,	concentration [$\text{g} \cdot \text{cm}^{-3}$];	Sc ,	Schmidt number defined in equation (15);
c_∞ ,	concentration in the bulk fluid [$\text{g} \cdot \text{cm}^{-3}$];	Sh ,	average Sherwood number;
D ,	diffusivity [$\text{cm}^2 \text{s}^{-1}$];	α ,	semi-vertex angle;
F, G, H ,	dimensionless velocity components defined in equations [3];	β ,	transformation variable, equation (8a), [$\text{cm}^{(4+2n)/(1+n)}$];
j ,	local mass flux [$\text{g} \cdot \text{cm}^{-2} \text{s}^{-1}$];	θ ,	tangential coordinate;
J ,	average mass flux [$\text{g} \cdot \text{cm}^{-2} \text{s}^{-1}$];	ξ ,	non-dimensional transformation variable defined in equation (4);
K ,	power law index [$\text{g} \cdot \text{cm}^{-1} \text{s}^{n-2}$];	ρ ,	density [$\text{g} \cdot \text{cm}^{-3}$];
L ,	cone slant length [cm];	ω ,	angular velocity [s^{-1}].
n ,	power law index;		
r ,	radial co-ordinate [cm];		
S ,	transformation variable, equation (10) [$\text{cm}^{(4+2n)/(1+n)}$];		
u ,	velocity component parallel to cone surface [$\text{cm} \text{s}^{-1}$];		
v ,	velocity component tangential to cone surface [$\text{cm} \text{s}^{-1}$];		
w ,	normal component of velocity [$\text{cm} \text{s}^{-1}$];		
x ,	co-ordinate parallel to cone surface [cm];		
y ,	co-ordinate normal to cone surface [cm];		

INTRODUCTION

IT HAS been demonstrated that the addition of small amounts of a long chain polymer to a fluid renders the fluid non-Newtonian and results in a significant reduction in the heat, mass and momentum transferred. The present study considers the problem of mass transfer in non-Newtonian solutions with particular application to rotating cones. Measurements were made of the transport of oxygen to the rotating surface and compared with theoretical results

which were obtained for high Schmidt numbers in laminar flow.

Two rotating disc studies are pertinent to the present investigation. In [1], Hansford and Litt reported on the mass transfer from a rotating disc made of the solute diffusing to non-Newtonian solutions. They measured the rates of dissolution of benzoic acid and β -naphthol into an aqueous solution of carboxymethylcellulose and of benzoic acid into aqueous polyethylene oxide. The value of the diffusivity was then determined by comparing their experimental results with a theoretical expression for the mass flux to a rotating disc in a non-Newtonian medium. Greif, Cornet and Kappesser [2] used a rotating disc system to determine the molecular diffusivity of dissolved oxygen in an aqueous sodium chloride solution. The solution was rendered non-Newtonian by adding various concentrations of Polyox WSR 301 (Union Carbide), a completely water soluble polymer of ethylene oxide.

THEORY

A fluid is said to be Newtonian when the stress components are linear functions of the rate-of-strain components. For a simple shearing motion the relation is then given by

$$\tau_{xy} = -\mu \frac{\partial u}{\partial y} \quad (1)$$

where the viscosity μ is a function of the temperature and the pressure but is not dependent on the rate of strain. The class of fluids which do not obey equation (1) is quite large; the most tractable belonging to the time independent pseudoplastic group which includes the polymer solutions of interest to us. For this group a "power law model" is used, and confirmed experimentally, which satisfies the following expression:

$$\tau_{xy} = -K \left(\frac{\partial u}{\partial y} \right)^n \quad (2)$$

The problem of interest to us is the mass transfer to a rotating cone in a quiescent fluid.

The laminar boundary-layer equations for the equations of continuity and the conservation of momentum have been given by Mitschka and Ulbrecht [3, 4] and will not be repeated here.

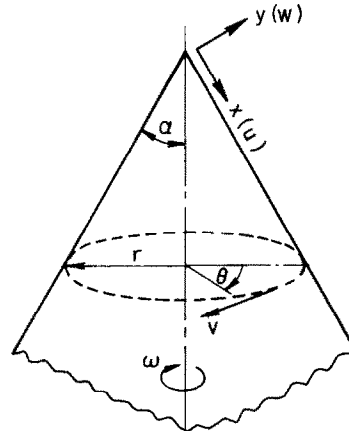


FIG. 1. Coordinate system.

The results for the velocity components u , v and w (cf. Fig. 1) are given by:

$$u = x\omega \sin \alpha F(\xi) \quad (3a)$$

$$v = x\omega \sin G(\xi) \quad (3b)$$

$$w = [x^{n-1} (\omega \sin \alpha)^{2n-1} K/\rho]^{1/(n+1)} H(\xi) \quad (3c)$$

where the similarity variable, ξ , is defined by

$$\xi = y \left[\frac{x^{1-n} (\omega \sin \alpha)^{2-n}}{K/\rho} \right]^{1/(1+n)} \quad (4)$$

Numerical values for the functions F , G and H and their derivatives have been tabulated by Mitschka and Ulbrecht for values of n between 0.2 and 1.5.

The diffusion equation is given by

$$\frac{u\partial c}{\partial x} + \frac{w\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \quad (5)$$

where c is the concentration of the diffusing species. For our system we note that the Schmidt number is large so that an asymptotic solution to the diffusion equation may be obtained by using a linear relation for the dimensionless radial velocity $F(\xi)$. This has

been successful in studying the non-Newtonian mass transfer to a rotating disc [1, 2, 5]. For the problem of Newtonian flow about a rotating cone at large Schmidt or Prandtl numbers reference should be made to Tien [6] and Sparrow and Gregg [7]. We therefore take $F = a\xi$ where $a = F'(0)$ which is tabulated by Mitschka and Ulbrecht. From the equation of continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{u}{x} = 0, \tag{6}$$

one obtains

$$H = -a \left(\frac{3+n}{2+2n} \right) \xi^2. \tag{7}$$

Using these results for F and H , or u and w , the diffusion equation becomes

$$\beta x^{2/(1+n)} y \frac{\partial c}{\partial x} - \beta x^{(1-n)/(1+n)} y^2 \left(\frac{3+n}{2+2n} \right) \frac{\partial c}{\partial y} = \frac{\partial^2 c}{\partial y^2} \tag{8}$$

where

$$\beta = \frac{a}{D} \left(\frac{\omega^3 \sin^3 \alpha}{K/\rho} \right)^{1/(n+1)}. \tag{8a}$$

This is the same equation that results for the mass transport to a rotating disc in non-Newtonian flow (5). The solution to equation (8) is given by

$$\frac{c}{c_\infty} = \frac{\gamma \left(\frac{1}{3}, \frac{S\beta}{3}, \frac{7+5n}{6+6n} \right)}{\Gamma(\frac{1}{3})} \tag{9}$$

where

$$S = y^3 x^{(1-n)/(1+n)}. \tag{10}$$

Γ is the gamma function, γ is the incomplete gamma function (8) and the boundary conditions

$$\begin{aligned} c &= 0 \text{ at } S = 0 \\ c &\rightarrow c_\infty \text{ as } S \rightarrow \infty \end{aligned} \tag{11}$$

are satisfied. The local mass flux at the surface,

$j = -D(\partial c/\partial y)_0$, may be obtained directly from equation (9). The quantity of interest to us is the average mass flux at the surface, J , which is given by

$$\begin{aligned} J &= \frac{\int_0^L j 2\pi x \sin \alpha \, dx}{\pi L^2 \sin \alpha} \\ &= \frac{3c_\infty D^{\frac{1}{3}} N(n) L^{\frac{1}{3}(1-n)/(1+n)} (\omega \sin \alpha)^{1/(1+n)}}{\Gamma(\frac{1}{3}) (K/\rho)^{1/[3(1+n)]}} \end{aligned} \tag{12}$$

where

$$N(n) = \left[12a \left(\frac{1+n}{7+5n} \right)^2 \right]^{\frac{1}{3}}. \tag{13}$$

Thus, the Sherwood number, Sh , becomes

$$Sh = \frac{JL}{Dc_\infty} \frac{3}{\Gamma(\frac{1}{3})} N(n) Sc^{\frac{1}{3}} Re_L^{(2+n)/[3(1+n)]} \tag{14}$$

with

$$Sc = \frac{K(\omega \sin \alpha)^{n-1}}{\rho D}, \quad Re_L = \frac{\rho L^2 (\omega \sin \alpha)^{2-n}}{K}. \tag{15}$$

EXPERIMENT

A schematic diagram of the experimental system is shown in Fig. 2. The vertical shaft was coupled to a variable speed drive. A monel cone with a plastic afterbody was mounted on the shaft and rotated in a container filled with an aerated 4% sodium chloride aqueous solution. Various concentrations of Polyox WSR 301, a completely water soluble polymer of ethylene oxide, were added to the solution, which resulted in a non-Newtonian fluid. The cone was made cathodic and the anode was a platinized titanium wire mesh cylinder of 20.3 cm dia. and 15 cm length.

The electrochemical system was operated galvanostatically. Cathode potentials relative to a saturated calomel cell were measured using a vacuum tube voltmeter and the current was increased in steps by using a variable resistor. From the inflection point of the resulting

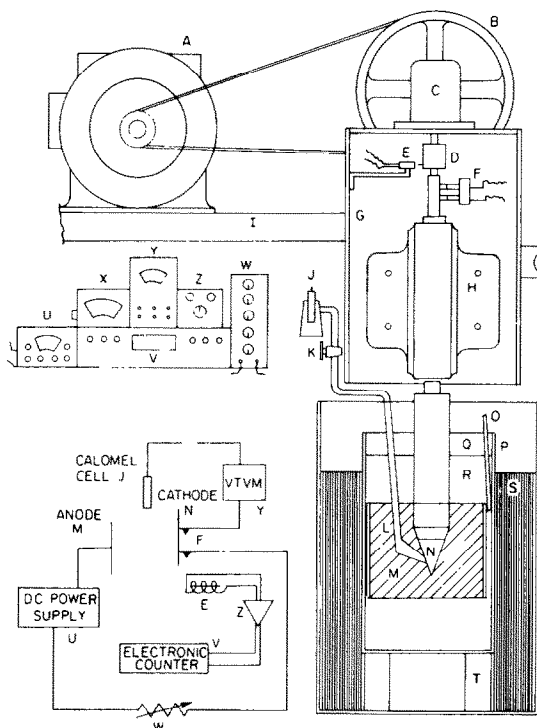
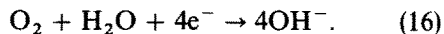


FIG. 2. Schematic diagram of the system.

- A. Varidrive
- B. Pulley and belt
- C. Miter box
- D. Coupling
- E. Magnetic pickoff
- F. Two carbon brushes
- G. Frame
- H. Pillow block
- I. Table
- J. Calomel cell
- K. Salt bridge (glass stopcock)
- L. Capillary probe
- M. Anode screen (platinized titanium)
- N. Specimen on shaft
- O. Thermometer
- R. 4% NaCl-water solution
- S. Tap water
- T. Metal spacer blocks
- U. D.C. power supply
- V. Electronic counter
- W. Variable resistor decade box
- X. Ammeter
- Y. Voltmeter
- Z. Amplifier

polarization curves the mass transfer limiting current was determined for the net reaction



Thus the measurement of the current allowed the mass transfer rate of oxygen diffusing to the surface to be computed. At cathode potentials above that for the oxygen mass transfer limiting current the net reaction $2\text{H}_2\text{O} + 2e^- \rightarrow 2\text{OH}^- + \text{H}_2$ resulted.

The cone tested was of 6.36 cm base diameter with a 60° vertex angle ($\alpha = 30^\circ$). The rotational speed was measured both by stopwatch and electronically, and the oxygen concentrations were measured by the Winkler technique. The power-law constants of the polymer solutions were determined from shear stress versus shear rate curves obtained from a Fann U-G rotating cylinder viscometer. The results are presented in Table 1.

Table 1. Viscometric properties of polyethylene oxide in 4% aqueous sodium chloride solution

Polyox concentration (wppm)	K ($\text{gcm}^{-1} \text{s}^{-n-2}$)	n	Temp. ($^\circ\text{F}$)
500	0.0689	0.81	77
3000	0.529	0.71	76
5000	1.405	0.65	75
10000	12.01	0.51	76

Experiments were carried out in four saline polymer solutions containing 500, 3000, 5000 and 10000 parts per million of Polyox WSR 301 by weight. For each solution the cone was rotated at speeds varying from approximately 2–1000 rpm. At each speed, mass transfer data was obtained from the polarization curves; a typical result being shown in Fig. 3. The current is the total current flowing between anode and cathode and the potential is relative to the calomel cell. The point of inflection exhibited by the polarization curve occurs at the mass transfer limiting current, this point corresponding to zero oxygen concentration at the metal surface. The mass flux of oxygen can then be calculated from the limiting current measure-

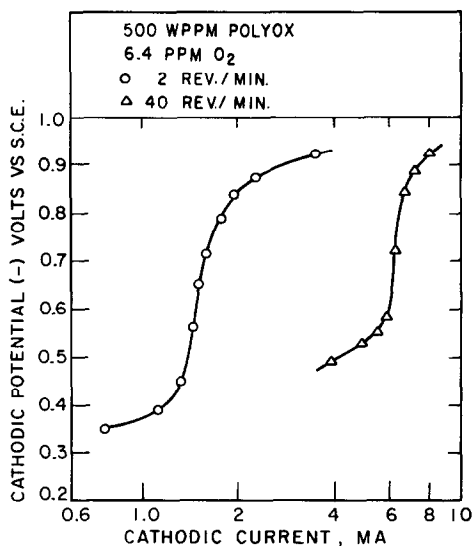


FIG. 3. Polarization curves.

ment in conjunction with equation (12). During each run several samples of solution were removed from the system and analyzed for oxygen by the Winkler technique.

For each of the solutions care had to be taken to ensure that no bubbles were present during the experimental run. This problem was particularly severe for the 10000 wppm Polyox solution. For this concentration it took three days for the bubbles to rise to the surface after mixing was completed. Furthermore, speeds in excess of 670 rpm could not be studied due to bubble entrainment.

The basic data is presented in Fig. 4 where the

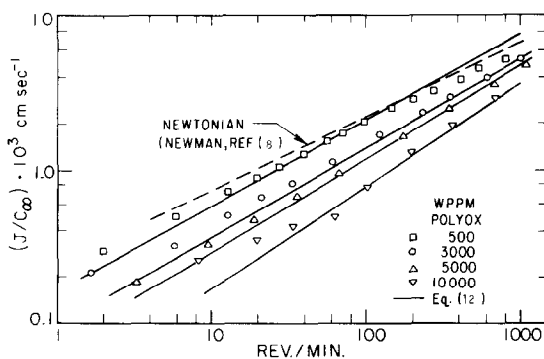


FIG. 4. Comparison of experimental data and theoretical prediction.

average mass transfer coefficient $J/\Delta c$ is plotted vs the cone angular velocity for each concentration of Polyox WSR 301.

DISCUSSION OF RESULTS AND CONCLUSIONS

As can be seen from Fig. 4 very good agreement was obtained between equation (12) and the experimental data. It should be emphasized that equation (12) contains no arbitrary constants and does not represent an empirical fit to the data. The values of diffusivity D and the rheological constants n and K were obtained *a priori* from mass transfer data for discs [2, 5] and viscometer measurements, respectively. Note, especially, the dependence of mass flux on RPM to the $1/n + 1$ power (cf. equation (12)).

For completeness, the theoretical result of Newman [8] is also presented in Fig. 4. This result is in excellent agreement with experimental data we have previously obtained in Newtonian flows.

For each of the polymer solutions the experimental data at low speeds was higher than the predicted values. This has been previously observed in Newtonian flows and has been attributed to the contribution from natural convection.

At high speeds another consideration should also be noted. Due to speed limitations on the Fann U-G viscometer, shear stress versus shear rate data could only be obtained for shear rates less than 960 s^{-1} . However, the maximum computed shear rates for the 500, 3000, 5000 and 10000 wppm concentrations of Polyox give values of 15200, 8100, 6800 and 2070 s^{-1} , respectively. This may be responsible for the departure of the mass transfer data for the 500 wppm Polyox solution at high rotational speeds—since it is known that under high rates of shear, pseudoplastic fluids cease to follow power law behavior and become Newtonian [9]. This is consistent with the trend of the data in Fig. 4 for the 500 wppm concentration. Note that the other concentrations, with their correspondingly

smaller rates of shear, follow the $(\text{rpm})^{1/n+1}$ dependence very closely.

Another effect which is of interest has been reported by Hansford and Litt [1] in their rotating disc study. They note that there is a departure from the $(\text{rpm})^{1/n+1}$ dependence at moderate rotational speeds which is due to the presence of secondary flows. The cone data in Fig. 4 for the 10000 wppm Polyox concentration shows a trend that is similar to that reported by Hansford and Litt. To explore this effect, small quantities of ink were injected into the fluid and it was found that for rotational speeds less than 100 rpm the flow field was divided into two distinct patterns. Away from the vertex, the effect of the centrifugal forces carries the flow towards the rear of the cone—as assumed in the theoretical development. However, near the vertex the flow was directed *towards* the vertex which is opposite to the expected centrifugal flow pattern.* At a rotational speed of 10 rpm the flow was almost entirely towards the vertex. However, as the speed was increased this reverse flow region decreased until, at 100 rpm, the reverse flow disappeared and the entire flow field became centrifugal. Note that the mass transfer data is in excellent agreement with the theoretical prediction for speeds of 100 rpm and greater for the 10000 wppm Polyox concentration.

In conclusion, the assumption of boundary layer flow and the linear approximation for the

* This has also been observed by Giesekus [10] for flows about cones although his test conditions differ from the present situation.

x velocity component appear to be justifiable. These assumptions yield a solution to the diffusion equation which is in good agreement with experimental data in the laminar flow regime for a 60° cone in a non-Newtonian fluid. For the 10000 wppm concentration Polyox, the divergence between the experimental and theoretical results for rotational speeds in the range between 10 and 100 rpm is due to the presence of a reverse flow.

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RESULTATS EXPERIMENTAUX ET THEORIQUES POUR UN TRANSFERT MASSIQUE A UN CONE TOURNANT DANS UNE SOLUTION SALINE NON-NEWTONIENNE

Résumé—On a obtenu une expression théorique pour le transport de chaleur et de masse à un cône tournant dans un fluide non-newtonien. Le fluide utilisé est une solution aqueuse de chlorure de sodium rendue non-newtonienne par addition de Polyox WSR 301. Les flux massiques mesurés expérimentalement pour un cône de 60° ont été comparés aux estimations théoriques et il a été trouvé un bon accord pour quatre solutions de polymères. On a observé pour la solution très visqueuse un écoulement secondaire à des vitesses de rotation modérées.

EXPERIMENTELLE UND THEORETISCHE ERGEBNISSE FÜR DEN STOFFÜBERGANG AN EINEN ROTIERENDEN KEGEL IN EINER NICHT-NEWTONISCHEN SALZLÖSUNG

Zusammenfassung—Der Wärme- und Stoffübergang an einen rotierenden Kegel in einer Nicht-Newtonischen Flüssigkeit wurde theoretisch erfasst. Die Flüssigkeit bestand aus wässriger Kochsalzlösung, die durch Zugabe von Polyose WSR 31 nicht-Newtonisch wurde. Der gemessene Stoffübergang an einen 60 °Kegel wurde mit den theoretischen Werten verglichen und für vier Polymerlösungen wurde gute Übereinstimmung festgestellt.

Für stark viskose Lösungen wurde bei geringen Rotationsgeschwindigkeiten eine Sekundärströmung festgestellt.

**ЭКСПЕРИМЕНТАЛЬНЫЕ И ТЕОРЕТИЧЕСКИЕ РЕЗУЛЬТАТЫ
ИССЛЕДОВАНИЯ ПЕРЕНОСА МАССЫ К ВРАЩАЮЩЕМУСЯ КОНУСУ В
НЕНЬЮТОНОВСКОМ СОЛЯНОМ РАСТВОРЕ**

Аннотация—Теоретически получено выражение для потока тепла и массы к вращающемуся конусу в неньютоновской жидкости. В качестве жидкости использовался водный раствор хлористого натрия, которому были приданы свойства неньютоновской жидкости путём добавления полиокса WSR 301. Экспериментально замеренные значения скорости переноса массы для конуса в 60° сравнивались с расчётными значениями. Получено хорошее соответствие между данными для четырёх полимерных растворов. При умеренных скоростях вращения в сильно вязком растворе наблюдалось вторичное течение.